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Playing with Truth, 2012
Cap 4

Assertoric Semantics and the Computational Power of Self-Referential Truth

Intro: acertijo (Laberinto, 1986)

Dos puertas, con un guardia en cada una.

Una lleva a la muerte segura, la otra permite salir del laberinto.

Un guardia siempre miente, un guardia siempre dice la verdad.

Sarah tiene sólo una pregunta para hacer.

Intro: acertijo (Laberinto, 1986)

Pregunta: si le preguntara a él si tu puerta es la salida, me diría que sí?

Hay 4 posibles pares (Guardia, Puerta), con:
G = {T = sincero, L = mentiroso) y
P = {M = muerte, S = salida}

(T, S) → R = No
(T, M) → R = Sí
(L, S) → R = No
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R = M, o sea, la rta. dice si la puerta del guardia interrogado es la de la muerte segura

Estructura

4.1 Abstract
4.2 La creencia “Mentiroso Inútil”
4.3 Semántica Asertórica
4.4 El poder computacional de la Verdad Auto-Referente (CPSRT)
4.5 Notas sobre la significación del CPSRT

Estructura

Cuestionar la creencia del Mentiroso Inutil

Mostrar que ciertas *query structures* (estructuras de consulta) pueden ser resueltas más eficientemente por lenguajes con recursos de auto referencia, usando el marco de trabajo de la *semántica asertórica.*

Se presentan las nociones de *complejidad de consulta clásica y mágica.*

Algunas reflexiones sobre las implicancias de este resultado en relación con las concepciones deflacionistas de la verdad.

La creencia “mentiroso inútil”

(L) L no es verdadero

ULC = El mentiroso (L) es inútil *dentro de* nuestro lenguaje

expresa una proposición? Infeliz, desafortunada, inútil?

Es útil, no sólo para desacreditar la teoría naïve de la Verdad, sino principalmente dentro del lenguaje.

La creencia “mentiroso inútil”

Usarla en forma interrogativa:

4.1 = Es L verdadera?

es una pregunta genuina, con una respuesta verdadera:

4.2 = 4.1 no puede ser contestada ni afirmativamente ni negativamente.

Mostrar que en un lenguaje con Mentirosos y Sinceros, la información puede ser obtenida más eficientemente que en uno sin recursos auto referentes.

La creencia “mentiroso inútil”

Assertoric formula: it is allowed to assert (deny) σ just in case by asserting (denying) σ one does not assert a falsehood, deny a truth, or contradict oneself.

In general, the assertoric value of a sentence is determined by two factors: by the **world** on the one hand and by the **assertoric rules** of one’s language on the other.

Roughly, the assertoric rules of a language are the **inferential rules**—for the logical constants, including the truth predicate, of the language—under an assertoric interpretation.

Con (L) => por las reglas asertóricas, no se puede ni afirmar ni negar, porque en ambos casos se llega a contradicción.

La creencia “mentiroso inútil”

En este punto menciona el acertijo, modificado para este problema:

4 banderas de diferentes colores: rojo, amarillo, verde o negro.

Tenemos una de las banderas, no sabemos el color, hay que hacerle preguntas al oráculo para saberlo.

El oráculo que siempre contesta con verdad preguntas del tipo sí/no (devuelve el valor asertórico, {0, 1}^2)

Con 2 preguntas clásicas se resuelve.

En lenguajes con recursos de auto referencia, se puede hacer con una sola.

La creencia “mentiroso inútil”

Pregunta:

- *Is it the case that: (L is not true and the flag is red) or (T is true and the flag is yellow) or (the flag is green)?*

Thus, though asking the Liar question by itself is not useful, within a language, that is in combination with other questions, the Liar can be used to create very efficient questions.

Semática Asertórica

Quotational closures and truth languages

Definition 4.1 Quotational closures and their classical fragment:

A quotational closure L has {∨, ∧, ¬, T } as its logical symbolism, where ‘T ’ is a unary truth predicate. Its non-logical symbolism consists, amongst others, of:

1. A set P = {p1 , p2 , . . .} of propositional atoms.
2. A finite set C = {c1 , c2 , . . . , cn } of non-

Semática Asertórica

Besides the non-quotational constant symbols C, L also contains a set of quotational constant symbols [C] = Con(L) – C.

The set Con(L), consisting of all the constant symbols (quotational or not) of L is jointly defined with Sen(L), the set of sentences of L. Sen(L) and Con(L) are the smallest sets satisfying:

$P \subseteq Sen(L), C \subseteq Con(L).$

$\alpha \in Sen(L) \Rightarrow [\alpha] \in Con(L).$

$t \in Con(L) \Rightarrow T(t) \in Sen(L).$

Semática Asertórica

Definition 4.2 Truth languages, their classical fragment and worlds

A truth language $\mathbf{L} = \langle L, \pi \rangle$ is a pair consisting of a quotational closure L and a function

$\pi : Con(L) \rightarrow Sen(L)$,

called a reference list, which satisfies

$\pi([\sigma]) = \sigma$ for all $\sigma \in Sen(L)$

Semática Asertórica

Throughout the paper, ‘λ’ and ‘τ ’ will be used as non-quotational constant symbols satisfying:

(4.3) $\pi(\lambda) = \neg T(\lambda)$, $\pi(\tau) = T(\tau)$

Under this convention, $\neg T(\lambda)$ models the Liar, whereas $T(\tau)$ models the Truthteller.

Semática Asertórica

An assertoric valuation function is a function

$V : Sen(L) \times W \rightarrow \{0, 1\}^2$,

taking a sentence σ and a world w as input and returning an element of $\{0, 1\}^2$, which is interpreted as the assertoric value of σ in w.

That is, with $V(\sigma, w) = (x, y)$, we have that:

$x = 1 \Leftrightarrow$ the assertion of σ is allowed in w .

$y = 1 \Leftrightarrow$ the denial of σ is allowed in w.

Semática Asertórica

A sentence $\sigma \in Sen(L)$ has either one of the following five forms:

p,

$(\alpha \wedge \beta)$,

$(\alpha \vee \beta)$,

$\neg\alpha$ or

$T(t)$.

The form of an assertoric sentence $X\sigma$ is specified by its sign, $X \in \{A, D\}$, and by the

Semática Asertórica

Depending on its type, an assertoric rule is depicted in either one of the following two ways.

$$\frac{X_\sigma}{\Pi(X_\sigma)}\sqcup \qquad \frac{X_\sigma}{\Pi(X_\sigma)}\sqcap$$

Semática Asertórica

Formally, the assertoric rule R(AF) associated with assertoric form AF can be thought of as a rule associating:

- each assertoric sentence of form AF
- with its set of immediate π sentences Π(Xσ), in either a conjunctive way (⊓) or in a disjunctive way (⊔).

The ten assertoric rules for L are, together with their type, displayed in the following table.

Semática Asertórica

$\dfrac{A_{\neg\alpha}}{\{D_\alpha\}}\sqcap$	$\dfrac{D_{\neg\alpha}}{\{A_\alpha\}}\sqcap$
$\dfrac{A_{(\alpha\vee\beta)}}{\{A_\alpha, A_\beta\}}\sqcup$	$\dfrac{D_{(\alpha\vee\beta)}}{\{D_\alpha, D_\beta\}}\sqcap$
$\dfrac{A_{(\alpha\wedge\beta)}}{\{A_\alpha, A_\beta\}}\sqcap$	$\dfrac{D_{(\alpha\wedge\beta)}}{\{D_\alpha, D_\beta\}}\sqcup$
$\dfrac{A_{T(t)}}{\{A_{\pi(t)}\}}\sqcap$	$\dfrac{D_{T(t)}}{\{D_{\pi(t)}\}}\sqcap$
$\dfrac{A_p}{\{A_p\}}\sqcap$	$\dfrac{D_p}{\{D_p\}}\sqcap$

Semática Asertórica

Definition 4.3 The $\mathcal{L}(\Rightarrow)$, $\mathcal{L}(\Leftarrow)$ and $\mathcal{L}(\Leftrightarrow)$ rules
Let AF be an assertoric form and let $\mathcal{R}(\mathsf{AF})$ be the associated assertoric rule. Depending on the type, $\sqcup$ or $\sqcap$, of $\mathcal{R}(\mathsf{AF})$, *the* $\mathcal{L}(\Rightarrow)$ *rule* associated with $\mathcal{R}(\mathsf{AF})$ is as follows. For any X_σ of form AF:

$\sqcap$: one is committed to X_σ $\Rightarrow$ one is committed to Y_α for all $Y_\alpha \in \Pi(X_\sigma)$.

$\sqcup$: one is committed to X_σ $\Rightarrow$ one is committed to Y_α for some $Y_\alpha \in \Pi(X_\sigma)$.

Semática Asertórica

As an example, consider the rule associated with $A_{\neg\alpha}$. As $A_{\neg\alpha}$ has type $\sqcap$ and as $\Pi(A_{\neg\alpha}) = \{D_\alpha\}$, the $\mathcal{L}(\Rightarrow)$ rule associated with the assertion of a negation reads as follows:

$$\text{one is committed to } A_{\neg_\alpha} \Rightarrow \text{one is committed to } D_\alpha.$$

Semática Asertórica

Definition 4.4 Validity of $\mathcal{L}(\cdot)$ rules with respect to $\mathcal{V}$
Let $\mathcal{V} : Sen(\mathcal{L}) \times W \to \{0,1\}^2$. We say that the $\mathcal{L}(\Rightarrow)$ *rules are valid with respect to* $\mathcal{V}$ just in case we have, for each assertoric sentence $X_\sigma \in \mathcal{X}$ and for each world $w \in W$ that:

$$\text{type of } X_\sigma \text{ is } \sqcap\text{: } \mathcal{V}_X(\sigma, w) = 1 \Rightarrow \mathcal{V}_Y(\alpha, w) = 1 \text{ for all } Y_\alpha \in \Pi(X_\sigma). \tag{4.4}$$

$$\text{type of } X_\sigma \text{ is } \sqcup\text{: } \mathcal{V}_X(\sigma, w) = 1 \Rightarrow \mathcal{V}_Y(\alpha, w) = 1 \text{ for some } Y_\alpha \in \Pi(X_\sigma). \tag{4.5}$$

Semática Asertórica

Definition 4.5 Branches of X_σ
A set $B \subseteq \mathcal{X}$ is a *branch* of X_σ just in case conditions 1, 2 , 3 and 4 hold.

1. $X_\sigma \in B$
2. $(Y_\alpha \in B$ and Y_α has type $\sqcap) \Rightarrow Z_\beta \in B$ for all $Z_\beta \in \Pi(Y_\alpha)$
3. $(Y_\alpha \in B$ and Y_α has type $\sqcup) \Rightarrow Z_\beta \in B$ for some $Z_\beta \in \Pi(Y_\alpha)$
4. For no $S \subset B$, condition 1,2 and 3 are satisfied.

Definition 4.6 Assertoric trees
Let $\sigma \in Sen(\mathcal{L})$. Its *assertion tree* $\mathfrak{T}^\sigma_A$ and its *denial tree* $\mathfrak{T}^\sigma_D$ are defined as follows:

$$\mathfrak{T}^\sigma_A = \{B \mid B \text{ is a branch of } A_\sigma\}$$

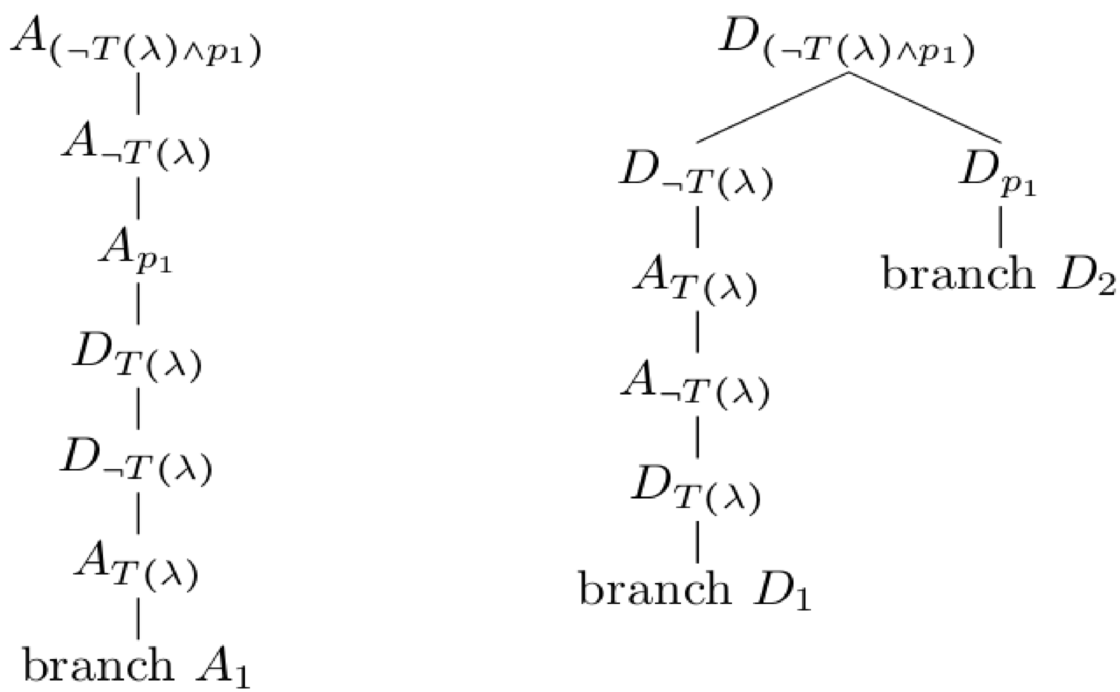
$$\mathfrak{T}^\sigma_D = \{B \mid B \text{ is a branch of } D_\sigma\}$$

An assertoric tree is thus a set of sets of assertoric sentences. □

Semática Asertórica

Although assertoric trees are officially sets of sets of assertoric sentences, we will depict them as "genuine trees". For instance, we do so in the following example.

Example 4.1 Assertoric trees
Below we depict $\mathfrak{T}_A^\gamma$ and $\mathfrak{T}_D^\gamma$, where $\gamma := (\neg T(\lambda) \wedge p_1) \in Sen(\mathcal{L})$.



Due to the finiteness of C, an assertoric tree is a *finite* object, being a finite set whose elements are finite sets of sentences. □

Semática Asertórica

By *closure conditions* for branches, we mean necessary and sufficient conditions which specify under which circumstances a branch is called *closed in a world* w, while a branch is called *open in* w just in case it is not closed in w. We write $C_w(B)$ and $O_w(B)$ as shorthand for 'branch B is closed in world w' and 'branch B is open in world w' respectively. *Closure conditions for branches define closure conditions for assertoric trees* according to the following schema.

$$C_w(\mathfrak{T}^\sigma_X) \Leftrightarrow_{def} C_w(B) \text{ for all } B \in \mathfrak{T}^\sigma_X \tag{4.6}$$

Equivalently, we can phrase this as:

$$O_w(\mathfrak{T}^\sigma_X) \Leftrightarrow_{def} O_w(B) \text{ for some } B \in \mathfrak{T}^\sigma_X \tag{4.7}$$

Semática Asertórica

Definition 4.7 $\mathcal{V}$ induced by closure conditions
Closure conditions for branches induce a valuation function $\mathcal{V} : Sen(\mathcal{L}) \times W \to \{0,1\}^2$. We define $\mathcal{V}$ in terms of its projector functions $\mathcal{V}_X$ as follows:

$$\mathcal{V}_X(\sigma, w) = 1 \Leftrightarrow O_w(\mathfrak{T}_X^\sigma),$$

where the closure conditions for trees derive from the closure conditions for branches as specified by equation (4.6) or (4.7). □

The Computational Power of Self-Referential Truth

Query structures, -strategies and -complexity

In a query structure, an agent wants to know the (classical) truth value (true or false) of all sentences in a set T ⊆ Sen(LP). Or, which is to say the same, the agent wants to decide his target knowledge T .

In order to decide T , the agent may invoke his background knowledge B ⊆ Sen(LP) in combination with additional information that he can gain by querying an oracle.

The Computational Power of Self-Referential Truth

Definition 4.10 Query structure
We say that a pair $\langle \mathcal{B}, \mathcal{T} \rangle$, consisting of $\mathcal{B} \subseteq Sen(L_P)$ and $\mathcal{T} \subseteq Sen(L_P)$, is a *query structure* just in case:

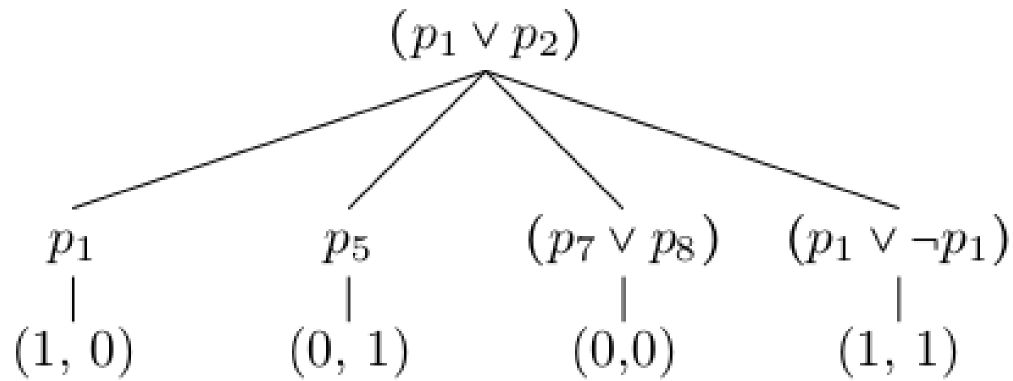
1. $\mathcal{B} \cup \mathcal{T}$ is consistent. $|\mathcal{T}|$ is finite.
2. $\exists \sigma \in \mathcal{T} : \mathcal{B} \nvdash_{L_P} \sigma$ and $\mathcal{B} \nvdash_{L_P} \neg\sigma$. (non-triviality)

Where a set $S \subseteq Sen(L_P)$ is consistent just in case there is a $\sigma \in Sen(L_P)$ such that $S \nvdash_{L_P} \sigma$. Here, $S \vdash_{L_P} \sigma$ means that $\sigma \in Sen(L_P)$ follows from $S \subseteq Sen(L_P)$ by propositional logic. □

We distinguish between two types of agents. A *classical agent* $\mathbf{A_{cla}}$ speaks L_P (and only L_P), while a *magical agent* $\mathbf{A_{ma}}$ speaks some[6] truth language $\mathcal{L}$. More concretely, $\mathbf{A_{cla}}$ can query the oracle by asking him questions in $Sen(L_P)$ while $\mathbf{A_{ma}}$ can ask the oracle questions in $Sen(\mathcal{L})$.

The Computational Power of Self-Referential Truth

Here is a 2-query strategy of $\mathbf{A}_{\mathbf{cla}}$.



In executing this strategy, $\mathbf{A}_{\mathbf{cla}}$ first asks the question $(p_1 \vee p_2)$ and then $\mathbf{A}_{\mathbf{cla}}$ asks a follow-up question which depends on the answer that the oracle gave to $(p_1 \vee p_2)$. Depending on whether the oracle answered $(p_1 \vee p_2)$ with $(1, 0)$, $(0, 1)$, $(0, 0)$ or $(1, 1)$ the agent respectively asks p_1, p_5, $(p_7 \vee p_8)$ or $(p_1 \vee \neg p_1)$ as a follow-up question. $\square$

The Computational Power of Self-Referential Truth

Definition 4.11 Solving a query structure
Let $\langle \mathcal{B}, \mathcal{T} \rangle$ be a query structure and let $\mathcal{S}$ be a query strategy of an arbitrary agent $\mathbf{A}$. We say that $\mathcal{S}$ *solves* $\langle \mathcal{B}, \mathcal{T} \rangle$ just in case[7] we have that, for each $w \in W$ and for each $\sigma \in \mathcal{T}$, after executing $\mathcal{S}$ in w, $\mathbf{A}$ can decide $\mathcal{T}$. Or, which is equivalent, we may say that $\mathcal{S}$ *solves* $\langle \mathcal{B}, \mathcal{T} \rangle$ just in case for every $w \in W$ we have that:

$$\forall \sigma \in \mathcal{T}\colon\ \mathcal{B} \cup \mathbf{KU}(\mathcal{S}, w) \vdash_{L_P} \sigma \text{ or } \mathcal{B} \cup \mathbf{KU}(\mathcal{S}, w) \vdash_{L_P} \neg\sigma \qquad \square$$

Definition 4.12 Query complexity, classical and magical
The *classical query complexity* $\mathcal{C}_{\langle \mathcal{B}, \mathcal{T} \rangle}$ of a query structure $\langle \mathcal{B}, \mathcal{T} \rangle$ is the least n for which $\mathbf{A_{cla}}$ has a query strategy $\mathcal{S}$ which solves $\langle \mathcal{B}, \mathcal{T} \rangle$.

The *magical query complexity* $\mathcal{M}_{\langle \mathcal{B}, \mathcal{T} \rangle}$ of a query structure $\langle \mathcal{B}, \mathcal{T} \rangle$ is the least n for which some[8] $\mathbf{A_{ma}}$ has a query strategy $\mathcal{S}$ which solves $\langle \mathcal{B}, \mathcal{T} \rangle$. $\square$

The Computational Power of Self-Referential Truth

Theorem 4.1 The Computational Power of Self-Referential Truth
Self-referential truth has computational power, as:

There exist query structures $\langle \mathcal{B}, \mathcal{T} \rangle$ such that: $\mathcal{M}_{\langle \mathcal{B}, \mathcal{T} \rangle} < \mathcal{C}_{\langle \mathcal{B}, \mathcal{T} \rangle}$

4.4.2 Knowledge updates from query strategies

In this section we define $\mathbf{KU}(\mathcal{S}, w)$ which is the knowledge update that an arbitrary agent $\mathbf{A}$ receives by executing query strategy $\mathcal{S}$ in w. Before we go over to the actual definition of $\mathbf{KU}(\mathcal{S}, w)$, we first present the *rationale* of this definition. Observe that:

1. By executing an n-query strategy $\mathcal{S}$ in w, $\mathbf{A}$ learns the answers of the oracle to n questions; say that $\mathbf{A}$ learns $\mathcal{V}^{\mathsf{as}}(\sigma_1, w), \ldots, \mathcal{V}^{\mathsf{as}}(\sigma_n, w)$.

2. Learning the assertoric value $\mathcal{V}^{\mathsf{as}}(\sigma, w)$ of σ is learning whether $O^{\mathsf{as}}_w(\mathfrak{T}^\sigma_A)$ or $C^{\mathsf{as}}_w(\mathfrak{T}^\sigma_A)$ and also learning whether $O^{\mathsf{as}}_w(\mathfrak{T}^\sigma_D)$ or $C^{\mathsf{as}}_w(\mathfrak{T}^\sigma_D)$.

3. Learning that $O^{\mathsf{as}}_w(\mathfrak{T}^\sigma_X)$ is learning that $O^{\mathsf{as}}_w(B)$ for some $B \in \mathfrak{T}^\sigma_X$. Learning that $C^{\mathsf{as}}_w(\mathfrak{T}^\sigma_X)$ is learning that $C^{\mathsf{as}}_w(B)$ for all $B \in \mathfrak{T}^\sigma_X$.

The Computational Power of Self-Referential Truth

A posteriori: hay un mundo donde un árbol asertórico está cerrado y otro donde está abierto.

A priori: no a posteriori.

The Computational Power of Self-Referential Truth

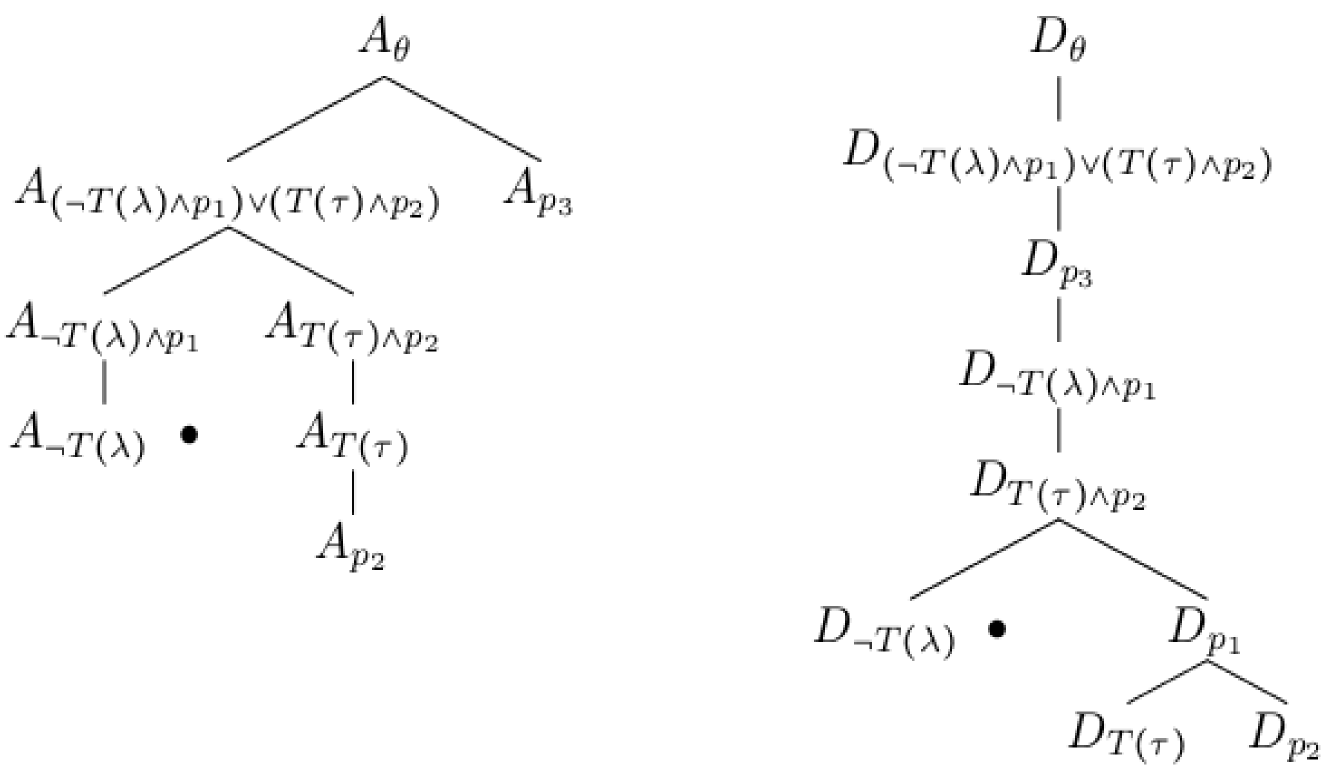
4.4.3 Magically, you can't do this classically

In this section we illustrate the computational power of self-referential truth. We do so by showing that the magical query complexity of the query structure 1-out-of-4 (see Example 4.3) is strictly less than its classical query complexity. Remember that, with θ_i as in Example 4.3, 1-out-of-4 = $\langle\{\theta_1 \vee \theta_2 \vee \theta_3 \vee \theta_4\}, \{p_1, p_2, p_3, p_4\}\rangle$, i.e. the agent's background knowledge is such that he knows that exactly 1 out of 4 given p_i is true and his target is find out which one it is. Clearly, $\mathbf{A_{cla}}$ can solve 1-out-of-4 in 2 questions and equally clearly, $\mathbf{A_{cla}}$ can not solve 1-out-of-4 in 1 question. Hence, the classical query complexity of 1-out-of-4 is 2. However, the magical query complexity of *1-out-of-4* is 1, as is illustrated by the strategy consisting of the single question θ.

$$\theta := ((\neg T(\lambda) \wedge p_1) \vee (T(\tau) \wedge p_2)) \vee p_3$$

The Computational Power of Self-Referential Truth

In order to illustrate that asking θ allows $\mathbf{A_{ma}}$ to decide his target, we display $\mathfrak{T}^{\theta}_{A}$ and $\mathfrak{T}^{\theta}_{D}$.[12]



Observaciones sobre la significación del CPSRT

Objeciones:

1) Esta noción de “complejidad de consulta mágica” no es una noción natural de complejidad de consulta.

2) Este resultado no tiene nada que ver con el poder computacional.

Observaciones sobre la significación del CPSRT

1) Esta noción de “complejidad de consulta mágica” no es una noción natural de complejidad de consulta. Se puede extender este 1-entre-4 a 1-entre-N, y esto hace que esta noción sea no natural. Si usarámos esta estrategia para, p ej., resolver consultas de 1-entre-6, se podría fabricar un modelo que lo cumpla, pero esta sistema de valores de 4 es de alguna manera intuitivo, en el sentido que se interpretan los 4 valores como el producto de “puedo afirmar” x “puedo negar”. En el caso de 6 sería más ad hoc.

Según el autor, esto no es un argumento en contra de la complejidad mágica.

Observaciones sobre la significación del CPSRT

2) Este resultado no tiene nada que ver con el poder computacional. La unidad de cómputo es el bit (0,1), y estos valores 4-valentes no es una manera natural de hablar de cómputo.

Rta: La noción de que la unidad de cómputo es el bit está desactualizada. Habla de computación cuántica, donde la unidad es el qubit. Los investigadores del área definieron la *complejidad de consulta cuántica.* También se puede resolver 1-entre-4 con computación cuántica.

Observaciones sobre la significación del CPSRT

Sería interesante cuestionar la complejidad en otros aspectos, como en espacio (memoria) o en tiempo.

Tal vez lo que ganamos en simplificación del query, lo perdemos en tiempo o espacio.

Por ejemplo, podemos estar exigiendo un oráculo (computadora) mucho más complejo que el clásico.

También está el asunto de la traducción que tiene que hacer el agente de las respuestas/oraciones entre los valores del oráculo y los valores de la consulta en consideración.

Observaciones sobre la significación del CPSRT

CPSRT y el deflacionismo: amigos o enemigos?

Parece ir en contra de las tendencias actuales deflacionistas predominantes.

Este trabajo podría usarse para probar que la concepción minimalista de la Verdad de Horwich es incorrecta:

- Todo el rol conceptual y teorético de la Verdad puede ser explicado sobre la base de todas las instancias no controversiales del esquema de equivalencia:
- (E) Es verdad que *p* si y sólo si *p.*

Si uno quiere admitir que la verdad tiene poder computacional, entonces este minimalismo no puede dar cuenta de esto, y necesitamos apelar a las reglas de inferencia de nuestro predicado de verdad.

Observaciones sobre la significación del CPSRT

CPSRT y el deflacionismo: amigos o enemigos?

Sin embargo, el CPSRT es compatible con algunas teorías deflacionistas: habría que comparar estos resultados con el *deflacionismo inferencial* de Horsten, que propone a la Verdad como una noción inferencial.

Además, esta Verdad es no-conservativa sobre las matemáticas y que "juega un papel más sustancial en ciertos debates filosóficos de lo que se esperaría a primera vista".

"Sólo hay leyes de la Verdad restringidas".

Si este deflacionismo es correcto, la Verdad tendrá poderes matemáticos y filosóficos, y además, poder computacional.

Observaciones sobre la significación del CPSRT

a quote from Grover—the philosopher, not the discoverer of the quantum search algorithm—illustrating a typical ULC attitude:

> [...] I give my reasons for holding the liar *is not* a sentence that is used in a *communicatively significant* way. This means that though the liar is syntactically well-formed, and its individual words have dictionary meaning, the liar *does not have "operative meaning"*. It is a sentence with *limited philosophical interest*. I recommend that our reaction to the liar should be *similar to our reaction to* 6/0. (Grover, [21, p178], my italics)

I hope to have convinced you that the Liar *is* a sentence that can be used in a *communicatively significant way* and that therefore the Liar *has "operative meaning"* and is of *great philosophical interest*. I recommend that our reaction to the Liar should be *similar to our reaction to* $\sqrt{-1}$: let us try to come up with a non-classical space in which we can make sense of the cognitive operations that are involved in our reflections on the Liar. If we succeed, it may be possible to assert that Grover is wrong because Grover is right.

Fin